Logs and Exponents

a) Prove that for $x > 1$:

$$a \int_{1/x}^{1} \frac{1}{t}\, dt = \int_{(1/x)^a}^{1} \frac{1}{t}\, dt.$$

b) Assume $x > 1$. What is the geometric interpretation of the result of part a?

c) What does this tell you about the area between the x-axis and the graph of $\frac{1}{x}$ over the interval from 0 to 1?

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a) Let $F(x) = \int_{1/x}^{1} \frac{1}{t}\, dt.$

$$a \int_{1/x}^{1} \frac{1}{t}\, dt = a\left(\ln(1) - \ln\left(\frac{1}{x}\right)\right)$$
$$= a(0 - 0 + \ln x)$$
$$= a \ln x$$
$$= \ln x^a$$

$$\Rightarrow \ln x^a = \ln \frac{1}{\frac{1}{x^a}}$$
$$= \ln 1 - \ln\left(\frac{1}{x^a}\right)$$
$$= \int_{(1/x)^a}^{1} \frac{1}{t}\, dt$$

b) The area of function $\frac{1}{x}$ is inversely proportional to $\frac{1}{x^a}$ by a constant a from $\frac{1}{x^a}$ to 1 for $\frac{1}{x} < 1$.

c) Decreasing the value of x decreases the distance of $\frac{1}{x}$ from 0 by $\sqrt[a]{x}$ over the interval from 0 to 1, which increases the area between the x-axis and the graph of $\frac{1}{x}$ by the constant a.